Study Pure Annihilation Decays $B_s^0(\bar{B}_s^0) \to D^{\pm}\pi^{\mp}$ in PQCD Approach*

Ying Li†, Cai-Dian Lü

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China

Institute of High Energy Physics, CAS, P.O.Box 918(4) Beijing 100049, China

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Abstract

The rare decays $B_s^0 \to D^{\pm}\pi^{\mp}$ and $\bar{B}_s^0 \to D^{\mp}\pi^{\pm}$ can occur only via annihilation type diagrams in the standard model. In this paper, we calculate branching ratios of these decays in perturbative QCD approach ignoring soft final state interaction. From our calculation, we find that their branching ratios are at $\mathcal{O}(10^{-6})$ with large CP asymmetry, which may be measured in LHC-b experiment in future.

1 Introduction

The rich data from two B factories make the study of B physics a very hot topic. A lot of study has been made, especially for the CP violation problem. The CKM angle $\beta = \phi_1$ has already been measured [1]. However the other two angles are difficult to measure in B factories. The study of B_s meson decay is needed for this purpose. Some work on B_s decays have already been done [2, 3].

In this work, we will explore four decay channels, namely $B_s^0 \to D^{\pm}\pi^{\mp}$ and $\bar{B}_s^0 \to D^{\mp}\pi^{\pm}$. There is only one kind of contribution for each of the decay mode, thus there is no direct

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†e-mail: liying@mail.ihep.ac.cn

CP violation for them. However there is still CP violation induced by mixing, although they are decays with charged final states (non CP eigenstates). They are quite complicated since altogether four are involved simultaneously [4].

From these decays, we find that the four quarks in final states are different from the ones in B_s^0 meson. We call this mode pure annihilation type decay. In the factorization approach, this decay is described as B_s^0 annihilating into vacuum and final states mesons produced from vacuum afterwards. They are rare decays. Up to now, only PQCD approach can calculate this kind of modes effectively. Using PQCD approach, we have calculated many of this kind of decays [3, 5, 6], and some decays have been measured in B factory. Some information about PQCD in detail can be found in ref.[7].

In standard model language, for decay $B_s \to D\pi$, a W boson exchange causes $\bar{b}s \to \bar{u}c$, and $\bar{d}d$ in final state are produced from a gluon. This is also called W exchange diagram. This gluon can be emitted by any one of quarks participating in the four quarks interaction. This is shown in Figure.1. We consider the B_s^0 meson at rest for simplicity. In this frame, this gluon has $\mathcal{O}(M_B/2)$ momenta, that's to say, this is a hard gluon. We can perturbatively treat the process by six quarks interaction.

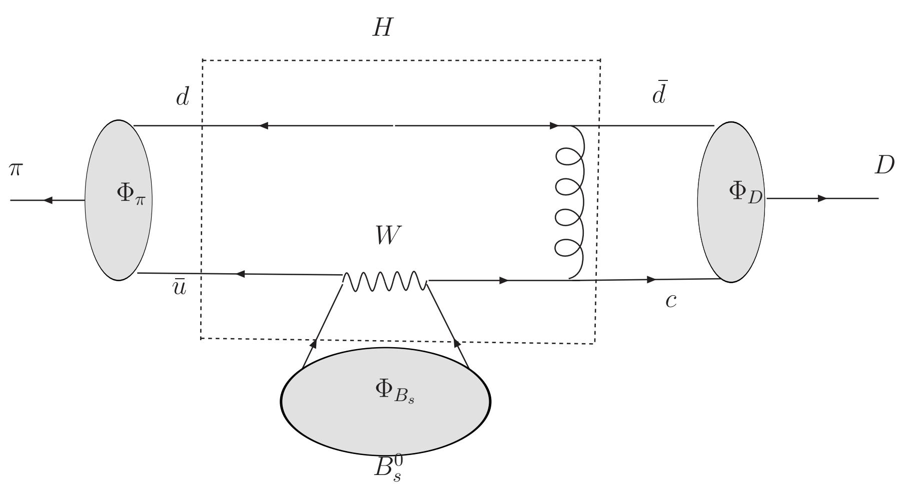

Figure 1: The picture of PQCD approach.

In this work, we will give the PQCD calculation of these two decays in the next section, and discuss the numerical results in section 3. At last we conclude this study in section 4.

2 Calculation

The non-leptonic B_s^0 decays $B_s^0 \to D^+\pi^-$ and $B_s^0 \to D^-\pi^+$ are rare decays. For decay $B_s^0 \to D^+\pi^-$, the effective Hamiltonian at the scale lower than M_W is given [8] as:

$$\mathcal{H}_1 = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} \Big[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \Big] , \tag{1}$$

where the four-quark operators are

$$O_1 = (\bar{b}s)_{V-A}(\bar{c}u)_{V-A} , \qquad\qquad O_2 = (\bar{b}u)_{V-A}(\bar{c}s)_{V-A} , \tag{2}$$

with the definition $(\bar{q}_1 q_2)_{V-A} \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. $C_{1,2}$ are Wilson coefficients at renormalization scale μ. For decay $B_s^0 \to D^-\pi^+$, the effective Hamiltonian read:

$$\mathcal{H}_2 = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} \Big[C_1'(\mu) O_1'(\mu) + C_2'(\mu) O_2'(\mu) \Big] , \tag{3}$$

where the four-quark operators are

$$O_1' = (\bar{b}s)_{V-A}(\bar{u}c)_{V-A} , \qquad\qquad O_2' = (\bar{b}c)_{V-A}(\bar{u}s)_{V-A} . \tag{4}$$

In PQCD, the decay amplitude is expressed as [7]:

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \ \mathrm{Tr}\big[C(t) \Phi_{B_s^0}(k_1) \Phi_D(k_2) \Phi_\pi(k_3) H(k_1,k_2,k_3,t) e^{-S(t)} \big]. \tag{5}$$

In this Equation, $C(t)$ is Wilson coefficient at scale t with leading order QCD correction. Φ_i are light-cone wave functions, which describe the non-perturbative contributions. They can not be theoretically calculated directly. Fortunately, they are process independent. $e^{-S(t)}$ is called Sudakov factor, which comes from the resummation of soft and collinear divergence. This Sudakov factor suppress the soft contributions, which make the perturbative calculation of hard part reliable. By including the k_T dependence of the wave functions and Sudakov form factor, this approach is free of endpoint singularity. Thus, the work left is calculating the perturbative hard part $H(t)$.

The structures of the meson wave functions are

$$B_s^0(P): \qquad [\not{P} + m_B]\gamma_5 \phi_B(x) , \tag{6}$$

$$D(P): \qquad \gamma_5[\not{P} + m_D]\phi_D(x) , \tag{7}$$

$$\pi(P): \qquad \gamma_5[\not{P}\phi_A(x) + m_0\phi_P(x) + m_0(\not{v}\ \not{n} - 1)\phi_T(x)] , \tag{8}$$

with $m_0 \equiv m_\pi^2/(m_u+m_d) = 1.4$ GeV, characterizing the chiral breaking scale. And the light-like vectors are defined as $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$. In above functions, ϕ_i are distribution amplitude wave functions.

According to the effective Hamiltonian (1), the diagrams contributing to $B_s^0 \to D^+\pi^-$ are drawn in Fig.2. Just as stated in Section 1, this decay has only annihilation diagrams. With the meson wave functions and Sudakov factors, the hard amplitude for factorizable annihilation diagrams in Fig.2(a) and (b) is,

$$
\begin{aligned}
F_a = \frac{64\pi}{3} M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2\, b_3 db_3\; \phi_D(x_2, b_2) \times \Big[\big\{ (1-x_3)\, \phi_\pi^A(x_3) \\
+ r\,(3-2x_3)\, r_\pi \phi_\pi^P(x_3) - r(1-2x_3) r_\pi \phi_\pi^T(x_3) \big\} E_f(t_a^1) h_a(x_2, x_3, b_2, b_3) \\
- \big\{ x_2 \phi_\pi^A(x_3) + 2r(1+x_2) r_\pi \phi_\pi^P(x_3) \big\} E_f(t_a^2) h_a(1-x_3, 1-x_2, b_3, b_2) \Big], \quad (9)
\end{aligned}
$$

where $r = m_D/M_{B_s^0}$, $r_\pi = m_0/M_{B_s^0}$, and the functions E_f containing Sudakov factors and Wilson coefficients of four quark operator, hard scale $t_a^{1,2}$ and virtual quark and gluon propagator h_a are given in the appendix. The explicit form for the distribution amplitude ϕ_M of wave functions, are given in the next section. In above function, x_i denotes light (anti-) quark momentum fraction in meson.

For the non-factorizable annihilation diagrams in Fig.2(c) and (d), the results can read:

$$
\begin{aligned}
M_a = \frac{256\pi}{3\sqrt{2N_c}} M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1\, b_2 db_2\; \phi_{B_s^0}(x_1, b_1) \phi_D(x_2, b_2) \\
\times \Big[\big\{ x_2 \phi_\pi^A(x_3, b_2) + r\,(1+x_2-x_3)\, r_\pi \phi_\pi^P(x_3, b_2) \\
+ r\,(1-x_2-x_3)\, r_\pi \phi_\pi^T(x_3, b_2) \big\} E_m(t_m^1) h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) \\
- \big\{ (1-x_3)\, \phi_\pi^A(x_3, b_2) + r\,(3+x_2-x_3)\, r_\pi \phi_\pi^P(x_3, b_2) \\
+ r\,(x_2-1+x_3)\, r_\pi \phi_\pi^T(x_3, b_2) \big\} E_m(t_m^2) h_a^{(2)}(x_1, x_2, x_3, b_1, b_2) \Big]. \quad (10)
\end{aligned}
$$

The total decay amplitude for $B_s^0 \to D^+\pi^-$ is given as

$$
A = f_B F_a + M_a. \quad (11)
$$

The decay width is expressed as

$$
\Gamma(B_s^0 \to D^+\pi^-) = \frac{G_F^2 M_B^3}{128\pi} (1-r^2) |V_{ub}^* V_{cs} A|^2. \quad (12)
$$

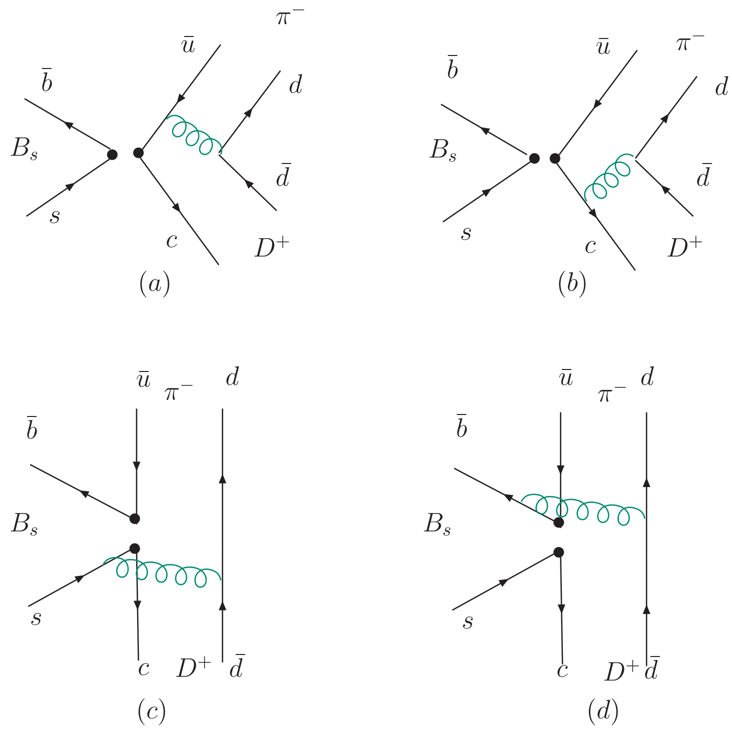

Figure 2: Leading order Feynman diagrams contributing to decay $B_s^0 \to D^+\pi^-$.

As the case $B_s^0 \to D^-\pi^+$, we also draw diagrams Fig.3 using Equation (3). The amplitude for factorizable annihilation diagrams (a) and (b) results in $-F_a$. The amplitude for the non-factorizable annihilation diagram results in

$$
\begin{aligned}
M'_a = \frac{256\pi}{3\sqrt{2N_c}} M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1\, b_2 db_2\ \phi_{B_s^0}(x_1, b_1)\phi_D(x_2, b_2) \\
\times \Big[\big\{(1-x_3)\,\phi_\pi^A(x_3, b_2) + r\,(x_2+1-x_3)\,r_\pi \phi_\pi^P(x_3, b_2) \\
+ r\,(x_2-1+x_3)\,r_\pi \phi_\pi^T(x_3, b_2)\big\} E_m(t_m^1) h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) \\
- \big\{x_2\phi_\pi^A(x_3, b_2) + r(3+x_2-x_3) r_\pi \phi_\pi^P(x_3, b_2) \\
+ r(1-x_2-x_3) r_\pi \phi_\pi^T(x_3, b_2)\big\} E_m(t_m^2) h_a^{(2)}(x_1, x_2, x_3, b_1, b_2)\Big]. \quad (13)
\end{aligned}
$$

Thus, the total decay amplitude A' and decay width Γ for $B_s^0 \to D^-\pi^+$ decay is given as

$$A' = -f_B F_a + M'_a, \quad (14)$$

$$\Gamma(B_s^0 \to D^-\pi^+) = \frac{G_F^2 M_B^3}{128\pi}(1-r^2)|V_{cb}^* V_{us} A'|^2. \quad (15)$$

The decays widths for CP conjugated mode, $\bar{B}_s^0 \to D^\mp\pi^\pm$, are the same expressions as $B_s^0 \to D^\pm\pi^\mp$, with the conjugate of CKM matrix elements.

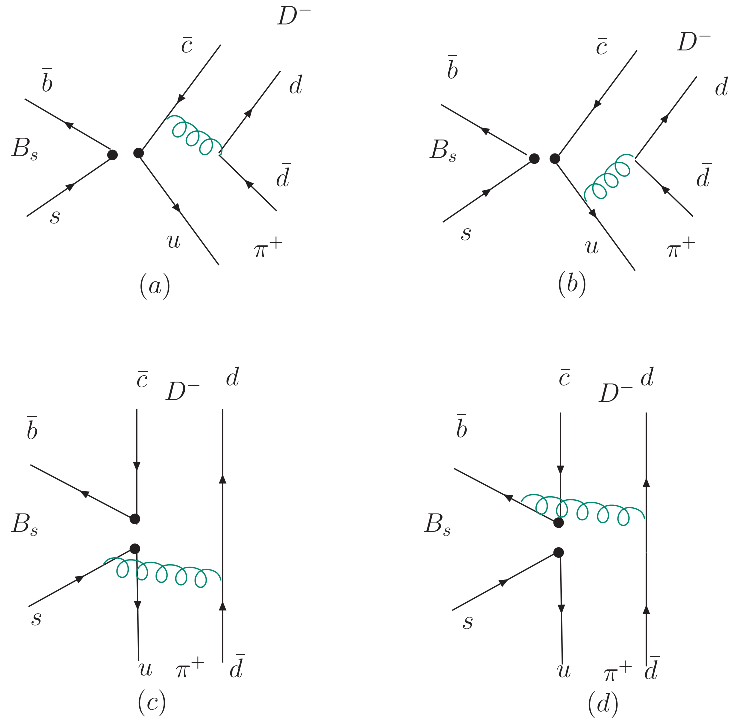

Figure 3: Leading order Feynman diagrams contributing to decay $B_s^0 \to D^-\pi^+$.

3 Numerical Evaluation

Considering SU(3) symmetry, we use the distribution amplitude of the B_s^0 meson similar to B meson:

$$\phi_{B_s^0}(x,b) = Nx^2(1-x)^2 \exp\left[-\frac{M_{B_s^0}^2\ x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \tag{16}$$

which is adopted in ref.[9, 10, 11]. N is a normalization factor, which can be get from normalized relation:

$$\int_0^1 dx\ \phi_M(x, b=0) = \frac{f_M}{2\sqrt{2N_c}}. \tag{17}$$

For D meson, the distribution amplitude is

$$\phi_D(x) = \frac{3}{\sqrt{2N_c}} f_D x(1-x)\{1 + a_D(1-2x)\}, \tag{18}$$

which is fitted from experiments [12]. The wave functions of the π meson have been derived in ref.[13, 14]:

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{2N_c}}x(1-x)\left[1+0.44C_2^{3/2}(2x-1)+0.25C_4^{3/2}(2x-1)\right] , \tag{19}$$

$$\phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}}\left[1+0.43C_2^{1/2}(2x-1)+0.09C_4^{1/2}(2x-1)\right] , \tag{20}$$

$$\phi_\pi^T(x) = \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)\left[1+0.55(10x^2-10x+1)\right] , \tag{21}$$

with the Gegenbauer polynomials,

$$\begin{aligned} C_2^{1/2}(t) &= \frac{1}{2}(3t^2-1) , \quad C_4^{1/2}(t) = \frac{1}{8}(35t^4-30t^2+3) , \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2-1) , \quad C_4^{3/2}(t) = \frac{15}{8}(21t^4-14t^2+1) . \end{aligned} \tag{22}$$

The other input parameters are listed below [15]:

$$\begin{aligned} &f_{B_s^0} = 230 \text{ MeV} , \ \ \omega_B = 0.5 \text{ GeV} , f_D = 240 \text{ MeV} , \ \ C_D = 0.8 \pm 0.2 , f_\pi = 132 \text{ MeV} , \\ &m_B = 5.37 \text{ GeV} , m_D = 1.87 \text{ GeV} , m_0 = 1.4 \text{ GeV} , \tau_{B^0} = 1.46 \times 10^{-12}\text{s} , \\ &|V_{cb}| = 0.043 , \ \ |V_{us}| = 0.22 , \ \ |V_{ub}| = 0.0036 , \ \ |V_{cs}| = 0.974 . \end{aligned} \tag{23}$$

With above parameters, we show the decay amplitudes calculated in Table.3. The predicted branching ratios are:

$$\text{Br}(B_s^0 \to D^+\pi^-) = 8.3 \times 10^{-7}, \tag{24}$$

$$\text{Br}(B_s^0 \to D^-\pi^+) = 2.9 \times 10^{-6}. \tag{25}$$

From above results, we find that the branching ratios of decay $B_s^0 \to D^+\pi^-$ is smaller than that of decay $B_s^0 \to D^-\pi^+$. The CKM element in decay $B_s^0 \to D^+\pi^-$ is $V_{ub}^*V_{cs}$, but in $B_s^0 \to D^-\pi^+$ the CKM element is $V_{cb}^*V_{us}$. Although $|V_{cb}^*V_{us}|$ and $|V_{ub}^*V_{cs}|$ are both $\mathcal{O}(\lambda^3)$ in Wolfenstein parametrization, the value of $\frac{|V_{ub}^*V_{cs}|}{|V_{cb}^*V_{us}|}$ is equal to 0.37. The branching ratio of $B_s^0 \to D^-\pi^+$ is 3 times larger than that of $B_s^0 \to D^+\pi^-$, which is mainly due to the CKM factor.

In addition to the perturbative annihilation contributions, there are another pictures existing such as soft final states interaction [16]. In ref.[5], the results from the PQCD approach for $B^0 \to D_s^- K^+$ is consistent with experiment well, which tells us the soft final

Table 1: Decay amplitudes (10^{-3} GeV) with parameters eqs. (9-13).

$B_s^0 \to D^+\pi^-$		$B_s^0 \to D^-\pi^+$	
$f_B F_a$	$0.51 - 1.3\,i$	$-f_B F_a$	$-0.51 + 1.3\,i$
M_a	$-16.1 - 19.1\,i$	M'_a	$-1.8 - 19.1\,i$
A	$-15.6 - 20.4\,i$	A'	$-2.3 - 17.8\,i$
Br	8.3×10^{-7}	Br	3.0×10^{-6}

states interaction may not be important. So we think their effects are small and ignore them in our calculation.

Unfortunately, there are no data for these two decays in experimental side up to now. We think that the LHC-b experiment can measure these decays in future. The results can test this PQCD approach and show some information about new physics.

The calculated branching ratios in PQCD approach are sensitive to various parameters such as the parameters in equations (16-23). The uncertainty taken by m_0 has been argued in many papers [10, 11], and it is strictly constrained by $B \to \pi$ form factor. In Table.2, we show the sensitivity of the branching ratios to change of B_s^0 and D distribution amplitude functions. It is found the uncertainty of the branching ratio in PQCD is mainly due to ω_b, which characterizes the shape of B_s^0 meson wave function.

Considering most of the uncertainty[1], we give the branching ratios of these two decays with suitable range of ω_b and a_D. Thus we can give our results:

$$\mathrm{Br}(B_s^0 \to D^+\pi^-) = (8.3\pm^{1.2}_{0.8}) \times 10^{-7} \left(\frac{f_{B_s}\ f_D}{230\ \mathrm{MeV} \cdot 240\ \mathrm{MeV}} \right)^2 \left(\frac{|V_{ub}^*\ V_{cs}|}{0.0036 \cdot 0.974} \right)^2, \tag{26}$$

$$\mathrm{Br}(B_s^0 \to D^-\pi^+) = (2.9\pm^{0.5}_{0.5}) \times 10^{-6} \left(\frac{f_{B_s}\ f_D}{230\ \mathrm{MeV} \cdot 240\ \mathrm{MeV}} \right)^2 \left(\frac{|V_{cb}^*\ V_{us}|}{0.0412 \cdot 0.224} \right)^2. \tag{27}$$

The CP violation information in decay $B_s(\bar{B}_s) \to D^{\pm}\pi^{\mp}$ is very complicated. There are four kinds of decays

$$\begin{aligned} g &= \langle D^+\pi^-|H|B_s^0\rangle \ \propto V_{ub}^* V_{cs}, \quad h = \langle D^+\pi^-|H|\bar{B}_s^0\rangle \ \propto V_{cb} V_{us}^*, \\ \bar{g} &= \langle D^-\pi^+|H|\bar{B}_s^0\rangle \ \propto V_{ub} V_{cs}^*, \quad \bar{h} = \langle D^-\pi^+|H|B_s^0\rangle \ \propto V_{cb}^* V_{us}, \end{aligned} \tag{28}$$

[1]Although the uncertainty taken by CKM matrix elements is large, we will not discuss them in this work, since they are only an overall factor here for branching ratios.

Table 2: The sensitivity of the branching ratios to change of ω_b and a_D

	(10^{-7})	(10^{-6})
ω_b	$\mathrm{Br}(B_s^0 \to D^+\pi^-)$	$\mathrm{Br}(B_s^0 \to D^-\pi^+)$
0.45	9.5	3.5
0.50	8.3	2.9
0.55	7.5	2.5
a_D	$\mathrm{Br}(B_s^0 \to D^+\pi^-)$	$\mathrm{Br}(B_s^0 \to D^-\pi^+)$
0.6	7.6	2.6
0.8	8.3	2.9
1.0	9.1	3.4

which determine the decay matrix element of $B_s^0 \to D^+\pi^-$ and $D^-\pi^+$, and of $\bar{B}_s^0 \to D^-\pi^+$ and $D^+\pi^-$. They are already shown in the previous Section. There is only one kind of contribution for each of the decay mode, thus there is no direct CP violation for them. However there is still CP violation induced by mixing, although they are decays with charged final states [4].

The time-dependent decay rates for $B_s \to D^\pm\pi^\mp$ are given by:

$$\begin{aligned}\Gamma^{D^\pm\pi^\mp}(t) &= (1 \pm A_{CP})\frac{e^{-t/\tau_{B_s}}}{8\tau_{B_s}}\{1 + (S_{D\pi} \pm \Delta S_{D\pi})\sin\Delta mt \\ &\quad +(C_{D\pi} \pm \Delta C_{D\pi})\cos\Delta mt\}\,,\end{aligned} \tag{29}$$

and $\bar{B}_s \to D^\pm\pi^\mp$ by

$$\begin{aligned}\bar{\Gamma}^{D^\pm\pi^\mp}(t) &= (1 \pm A_{CP})\frac{e^{-t/\tau_{B_s}}}{8\tau_{B_s}}\{1 - [(S_{D\pi} \pm \Delta S_{D\pi})\sin\Delta mt \\ &\quad +(C_{D\pi} \pm \Delta C_{D\pi})\cos\Delta mt]\}\,.\end{aligned} \tag{30}$$

Utilizing eq.(28), we can get

$$\begin{aligned} &C_{D\pi} = A_{CP} = 0, \qquad && \Delta C_{D\pi} = \frac{1-|h/g|^2}{1+|h/g|^2}, \\ &S_{D\pi} = \frac{2\,|h/g|\sin\gamma\cos\delta}{1+|h/g|^2}, \quad && \Delta S_{D\pi} = \frac{-2\,|h/g|\sin\delta\cos\gamma}{1+|h/g|^2}. \end{aligned} \tag{31}$$

In deriving the above formulas we have neglected the small weak phase $arg(q/p) = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} = 2\lambda^2\eta < 2^\circ$, in Wolfenstein parametrization [17].

We can calculate these parameters related to decays $B_s^0(\bar{B}_s^0) \to D^\pm\pi^\mp$ in our PQCD approach. Through calculation, we get:

$$\Delta C_{D\pi} = -0.56. \tag{32}$$

The parameters $S_{D\pi}$ and $\Delta S_{D\pi}$ are γ related. The results are shown in Figure.4. If we can measure the time-dependent spectrum of the decay rates of B_s^0 and $\bar{B}_s^0$, we can extract the CKM angle γ and strong phase δ in eq.(31) by Figure.4. The parameter $C_{D\pi} = A_{CP} = 0$ is from the fact that there is only one kind of contribution to each of the decays. Any deviation from zero, will be a signal of new physics contribution.

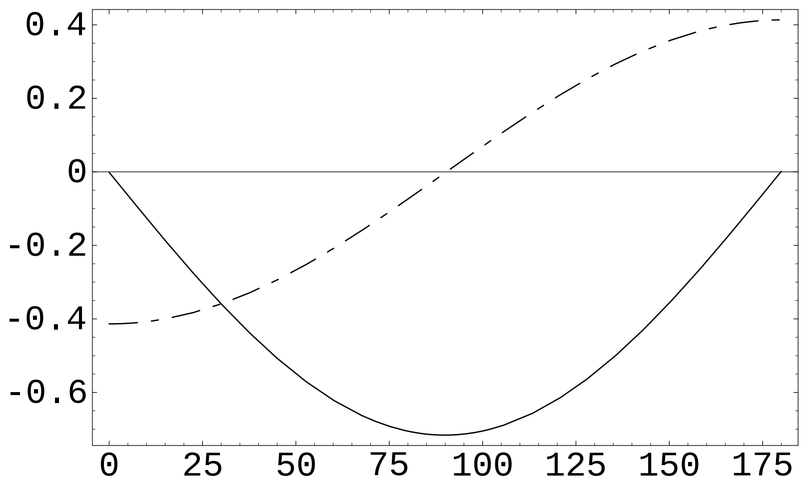

Figure 4: CP violation parameters of $B_s^0(\bar{B}_s^0) \to D^\pm\pi^\mp$: $\Delta S_{D\pi}$ (dash-dotted line) and $S_{D\pi}$ (solid line) as a function of CKM angle γ.

4 Summary

Recent study shows that PQCD approach works well for charmless B decays [10, 11], as well as for channels with one charmed meson in the final states [5, 12]. Because the final state mesons are moving very fast, each of them carrying more than 2 GeV energy, there is not enough time for them to exchange soft gluons. So we can ignore the soft final states

interaction. Due to disadvantages in other approach such as general factorization approach [18] and BBNS approach [19], pure annihilation decay can be calculated reliably only in PQCD approach.

In this paper, we calculate $B_s^0 \to D\pi$ decays, which occur purely via annihilation type diagrams. The branching ratios are still sizable at the order of 10^{-6}. There will also be sizable CP violation in these decays. They will be measured in future LHC-b experiment, which may bring some information about new physics to us.

A Some functions

The definitions of some functions used in the text are presented in this appendix. In the numerical analysis we use one loop expression for strong coupling constant,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)}, \tag{33}$$

where $\beta_0 = (33 - 2n_f)/3$ and n_f is number of active flavor at appropriate scale. Λ is QCD scale, which we use as 250 MeV at $n_f = 4$. We also use leading logarithms expressions for Wilson coefficients $C_{1,2}$ presented in ref.[8]. Then, we put $m_t = 170$ GeV, $m_W = 80.2$ GeV, and $m_b = 4.8$ GeV.

The function E_f^i, E_m, and E'_m including Wilson coefficients are defined as

$$E_f^i(t) = \left[C_1(t) + C_2(t)3\right]\alpha_s(t)\, e^{-S_D(t)-S_\pi(t)}, \tag{34}$$

$$E_m(t) = C_2(t)\alpha_s(t)\, e^{-S_B(t)-S_D(t)-S_\pi(t)}, \tag{35}$$

$$E'_m(t) = C_1(t)\alpha_s(t)\, e^{-S_B(t)-S_D(t)-S_\pi(t)}, \tag{36}$$

where S_B, S_D, and S_π result from summing both double logarithms caused by soft gluon corrections and single ones due to the renormalization of ultra-violet divergence. The above $S_{B,D,\pi}$ are defined as

$$S_B(t) = s(x_1 P_1^+, b_1) + 2\int_{1/b_1}^{t} \frac{d\mu'}{\mu'}\gamma_q(\mu'), \tag{37}$$

$$S_D(t) = s(x_2 P_2^+, b_3) + 2\int_{1/b_2}^{t} \frac{d\mu'}{\mu'}\gamma_q(\mu'), \tag{38}$$

$$S_\pi(t) = s(x_3 P_3^+, b_3) + s((1-x_3)P_3^+, b_3) + 2\int_{1/b_3}^{t} \frac{d\mu'}{\mu'}\gamma_q(\mu'), \tag{39}$$

where $s(Q,b)$, so-called Sudakov factor, is given as [22]

$$\begin{aligned} s(Q,b) &= \int_{1/b}^{Q} \frac{d\mu'}{\mu'} \left[\left\{ \frac{2}{3}(2\gamma_E - 1 - \log 2) + C_F \log \frac{Q}{\mu'} \right\} \frac{\alpha_s(\mu')}{\pi} \right. \\ &\quad \left. + \left\{ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3}\beta_0 \log \frac{\gamma_E}{2} \right\} \left(\frac{\alpha_s(\mu')}{\pi} \right)^2 \log \frac{Q}{\mu'} \right], \end{aligned} \tag{40}$$

$\gamma_E = 0.57722\cdots$ is Euler constant, and $\gamma_q = \alpha_s/\pi$ is the quark anomalous dimension.

The functions h_a, $h_a^{(1)}$, and $h_a^{(2)}$ in the decay amplitudes consist of two parts: one is the jet function $S_t(x_i)$ derived by the threshold resummation [20], the other is the propagator of virtual quark and gluon. They are defined by

$$\begin{aligned} & h_a(x_2,x_3,b_2,b_3) = S_t(1-x_3) \left(\frac{\pi i}{2} \right)^2 H_0^{(1)}(M_B \sqrt{(1-r^2)x_2(1-x_3)}\, b_2) \\ & \times \left\{ H_0^{(1)}(M_B \sqrt{(1-r^2)(1-x_3)}\, b_2) J_0(M_B \sqrt{(1-r^2)(1-x_3)}\, b_3) \theta(b_2 - b_3) + (b_2 \leftrightarrow b_3) \right\}, \end{aligned} \tag{41}$$

$$\begin{aligned} & h_a^{(j)}(x_1,x_2,x_3,b_1,b_2) = \\ & \left\{ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B \sqrt{(1-r^2)x_2(1-x_3)}\, b_1) \mathrm{J}_0(M_B \sqrt{(1-r^2)x_2(1-x_3)}\, b_2) \theta(b_1 - b_2) \right. \\ & \qquad \left. + (b_1 \leftrightarrow b_2) \right\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{(j)} b_1), & \text{for} \quad F_{(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B \sqrt{|F_{(j)}^2|}\, b_1), & \text{for} \quad F_{(j)}^2 < 0 \end{pmatrix}, \end{aligned} \tag{42}$$

where $\mathrm{H}_0^{(1)}(z) = \mathrm{J}_0(z) + i\,\mathrm{Y}_0(z)$, and $F_{(j)}$s are defined by

$$F_{(1)}^2 = (1-r^2)(x_1 - x_2)(1-x_3), \; F_{(2)}^2 = x_1 + x_2 + (1-r^2)(1-x_1-x_2)(1-x_3). \tag{43}$$

We adopt the parametrization for $S_t(x)$ of the factorizable contributions,

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)} [x(1-x)]^c, \quad c = 0.3, \tag{44}$$

which is proposed in ref. [21]. In the non-factorizable annihilation contributions, $S_t(x)$ gives a very small numerical effect to the amplitude. Therefore, we drop $S_t(x)$ in $h_a^{(1)}$ and $h_a^{(2)}$. The hard scale t's in the amplitudes are taken as the largest energy scale in the H to kill the large logarithmic radiative corrections:

$$t_a^1 = \max(M_B \sqrt{(1-r^2)(1-x_3)}, 1/b_2, 1/b_3), \tag{45}$$

$$t_a^2 = \max(M_B \sqrt{(1-r^2)x_2}, 1/b_2, 1/b_3), \tag{46}$$

$$t_m^j = \max(M_B \sqrt{|F_{(j)}^2|}, M_B \sqrt{(1-r^2)x_2(1-x_3)}, 1/b_1, 1/b_2). \tag{47}$$

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